

## Chapter 2

1. A first way of answering the question is to point out that  $\theta_p$ , the person parameter, is a person-specific intercept. A second way of answering is to point out that there is an alternative formulation of the model that would contain an intercept  $\beta_0$  indeed. Suppose that we use only I-1 item predictors, one for each item but, for example, not for the I-th. The I-th predictor would then be a vector with ones. Following this parameterization, the resulting item parameters ( $\beta'_i$ ) can be mapped into the previous ones ( $\beta_i$ ) as follows:  $\beta_i = \beta'_i + \beta_0$ , but  $\beta_I = \beta_0$  (because  $\beta'_I = 0$ ), and the resulting person parameters ( $\theta'_p$ ) can be mapped into the previous ones ( $\theta_p$ ) as follows:  $\theta_p = \theta'_p + \beta_0$ .

2. In this new model, the roles of the persons and the items are interchanged. As for the persons, there would be P person predictors, with  $Z_{pk}$  being the value of person p on person predictor k ( $k = 1, \dots, P$ ), so that  $Z_{pk} = 1$  if  $p = k$ , and  $Z_{pk} = 0$  otherwise. The weights of the  $Z_{pk}$  would be fixed person parameters  $\theta_{p=k}$ . As for the items, instead of having I item predictors, there would be only one,  $X_{i0}$ , with  $X_{i0} = 1$  for all values of i. The weights of this predictor would be the random effects  $\beta_i$ . As a consequence,  $X_{ik}$  would be replaced with  $X_{i0}$ ,  $\beta_i$  would be put in a dotted circle,  $Z_{p0}$  would be replaced with  $Z_{pk}$ , and  $\theta_p$  would be put in a circle with a full line.

3. The intercept of the LLTM depends on the way the item properties are coded, just as in a regular regression model. Because the overall intercept  $\beta_0$  does not depend on the item, one can add this intercept to  $\theta_p$  in order to obtain a new person specific intercept  $\theta'_p$  (see Exercise 1). If  $\beta_0$  is fixed to zero, then one should estimate the mean of the  $\theta$  distribution, which would then be  $\beta_0$ . If the mean of  $\theta$  is a free parameter, then this mean is the  $\beta_0$  from a model formulation with a free intercept and a zero mean of  $\theta$ .

4. The estimates of Situation Type, Behavior Type (predictors 2, 3, 4) do not change, and also the estimated variance of  $\theta$  is the same. Two other estimates are different. The estimated effect of Do vs Want now is .34 (SE=.03), which is half the effect that is estimated with dummy coding. This is because the difference in the coding is now 2 ( $1 - (-1)$ ) instead of 1 ( $1 - 0$ ). Also the estimate of the intercept differs. It now has a value of .65 (SE=.09), the original .31 plus half the earlier effect of Do vs Want (.67/2).

5. The new figure would be one with (a) the arrow from the ellipse with the  $\vartheta$  pointing directly to the dotted circle with  $\eta_{pi}$ , (b) the arrow from the dotted circle with  $\varepsilon$  pointing directly to the dotted circle with  $\eta_{pi}$ . Yes,  $\varepsilon_p$  would be the random intercept. The error term would reflect the unexplained part of the individual differences. One can technically call this an error term because it reflects the errors in the prediction. However, when the prediction model is incomplete, it may as well reflect systematic variance.