1

Solutions to the exercises of chapter 8

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1.1 Exercise 1

The model of the form $\eta_{pi} = \theta_{i1} + \theta_{i2} - \beta_i$ is not identified. The two dimensions are identical: all items measure the two dimensions to the same extent. Only their sum is identified.

1.2 Exercise 2

Let $\beta_i^* = \beta_i/\alpha_i$. Equation 8.5 then becomes $\eta_{pi} = \alpha_i\theta_p - \alpha_i\beta_i^* = \alpha_i(\theta_p - \beta_i^*)$. Adaptation of the SAS-code is straightforward.

1.3 Exercise 3

The code to estimate the model is exactly the same as the one given in Section 8.5.1 for the between-item two-dimensional model without latent item predictors, except for the fact that the first dimension is now common to all items. Hence, one has to replace "eta = Want*th_d + ..." with "eta = th_1 + ...".

The deviance of the resulting model is 7980. The estimates and standard errors of the item parameters are given in Table 8A.1. With an eye to Exercise 4, the items have the same order as for the between-item model discussed in Section 8.2.4. The estimates of the standard deviations for the common dimension and the dimension corresponding to the do-items are 1.45 and 1.07 respectively, both with a standard error of 0.09. The estimated correlation between the two dimensions is -.14, with a standard error of 0.10.

TABLE 1.1. Estimates and standard errors of the item parameters for the whithin-item two-dimensional model with no latent item predictors of Exercise 8.3.

Situation	Behavior Type	Behavior Mode	β_i	s.e.(β_i)
train	curse	want	-1.79	0.18
bus stop	curse	want	-1.25	0.17
call	curse	want	-1.11	0.16
train	scold	want	-0.73	0.16
bus stop	scold	want	-0.58	0.16
store	curse	want	-0.55	0.16
bus stop	shout	want	-0.09	0.16
train	shout	want	-0.02	0.16
call	scold	want	0.35	0.16
store	scold	want	0.70	0.16
call	shout	want	1.06	0.16
store	shout	want	1.56	0.18
bus stop	curse	do	-1.32	0.18
train	curse	do	-0.94	0.17
call	curse	do	-0.75	0.17
bus stop	scold	do	-0.40	0.17
train	scold	do	0.09	0.17
store	curse	do	0.25	0.17
call	scold	do	0.44	0.17
bus stop	shout	do	0.97	0.17
train	shout	do	1.62	0.18
store	scold	do	1.65	0.19
call	shout	do	2.17	0.20
store	shout	do	3.21	0.25

1.4 Exercise 4

As was already hinted at in the chapter when comparing the between-item two-dimensional model with the Rasch model, the within-item two-dimensional model of Exercise 3 is formally equivalent to the between-item two-dimensional model discussed in Section 8.2.4. The models only differ in how the (same) latent space is taken into account by the respective models: a dimension common to all items and an additional dimension for the do-items, versus two separate dimensions for the want- and do-items. By consequence, the deviances for the two models are the same, and all parameter estimates are identical except for the parameters characterizing the distribution of the latent variables. As we show now, also the latter estimates can be translated from the one model into the other, so that running the analysis for the within-item two-dimensional model of Exercise 2 was actually superfluous. The latent variable part of the linear predictor of the between-item two-dimensional model is

$$want_i \, \theta_{pw} + Do_i \, \theta_{pd}, \tag{1.1}$$

which can be rewritten as

$$(Constant - Do_i) \, \theta_{pw} + Do_i \, \theta_{pd} = Constant \, \theta_{pw} + Do_i \, (\theta_{pd} - \theta_{pw}). \tag{1.2}$$

For the within-item two-dimensional, we have

$$constant \, \theta_{p1}{}^{*} + Do_i \, \theta_{pd}{}^{*}. \tag{1.3}$$

Hence, $\theta^*_{p1} = \theta_{pw}$ and $\theta^*_{pd} = \theta_{pd} - \theta_{pw}$. Using the identities,

$$\sigma^2_{X \pm Y} = \sigma^2_X + \sigma^2_Y \pm 2\sigma_{XY}, \tag{1.4}$$

$$\sigma_{X \pm Y, Y} = \sigma_{XY} \pm \sigma^2_Y, \tag{1.5}$$

the variances and covariance for the within-item two-dimensional model can be derived from the between-item two-dimensional model as follows:

$$\sigma^2_{\theta^*_1} = \sigma^2_{\theta_w}, \tag{1.6}$$

$$\sigma^2_{\theta_d{}^*} = \sigma^2_{\theta_d} + \sigma^2_{\theta_w} - \sigma_{\theta_d \theta_w} \tag{1.7}$$

$$\sigma_{\theta_1{}^* \theta_d{}^*} = \sigma_{\theta_w \theta_d} - \sigma^2_{\theta_w} \tag{1.8}$$

which can be easily checked using the results of Exercise 3.

1.5 Exercise 5

The code to estimate the RW-LLTM is the following (the code is the same as for the LLTM except for the inclusion of the term *th_o∗Other* :

```
PROC NLMIXED data=want_do method = gauss noad qpoints=20
    technique=newrap;
PARMS b1-b5 = 0 s_1 = 1 s_1o = 0 s_o = 1;
eta= th_1 + th_o*Other -(b1 + b2*Scold + b3*Curse + b4*Do + b5*Other);
expeta=exp(eta);
p=expeta/(1+expeta);
MODEL y~binary(p);
RANDOM th_1 th_o ~normal([0,0],[s_1**2, s_1o, s_o**2]) subject = pp;
estimate 'var_1' s_1**2;
estimate 'var_o' s_o**2;
estimate 'cor' s_1o/(s_1*s_o);
RUN;
```

The item properties were coded into four item predictors using dummy coding as follows:

Scold = 1/0 if Behavior Type = scolding/otherwise
Curse = 1/0 if Behavior Type = cursing/otherwise
Do = 1/0 if Behavior Mode = doing/wanting
Other = 1/0 if Situation Type = other-/self-to-blame.

The deviance of the RW-LLTM was 8162. The estimates and standard errors of the fixed weights are given in table 8A.2. We also included the results for the LLTM in a fourth and fifth column. The results indicate that cursing is the most likely behavior and shouting the least likely, that wanting is more likely than doing, and that one is more inclined to act in a verbally aggressive way in situations where another person is to blame. The estimates of the standard deviations for the common dimension and the dimension corresponding to the Other-to-blame-items are 1.28 and .093 respectively, with standard errors of 0.08 and 0.09, respectively. The estimated correlation between the two dimensions is .11, with a standard error of 0.12. To test for the need for a random slope for Other-vs. Self-to-blame, the LLTM has to be contrasted with the RW-LLTM. The asymptotic null distribution of the likelihood ratio test-statistic is a mixture with equal weights of .5 of two chi-squared distributions, respectively with two degrees and one degree of freedom (Verbeke, 1997; Chapter 4 of this volume). The likelihood-ratio test statistic amounted to $8232 - 8162 = 70, p < .001$, so that the LLTM is rejected. Nevertheless, the estimates for the weights of the item predictors were very similar under both models. In comparison with the estimates obtained for the RW-LLTM, the estimates for the

TABLE 1.2. Estimates and standard errors of the fixed weights for a) the RW-LLTM with a random slope for *Other* (Other- vs. Self-to-blame), and b) the LLTM.

	RW-LLTM		LLTM	
Item predictor	β_i	s.e.(β_i)	β_i	s.e.(β_i)
Constant	1.35	0.10	1.32	0.10
Scold	-1.04	0.07	-.99	0.07
Curse	-2.12	0.08	-2.04	0.07
Do	0.70	0.06	0.67	0.06
Other	-1.07	0.08	-1.03	0.06

LLTM model are slightly shrunk towards zero, similar to what was observed in Chapter 8 when comparing the Rasch model to the between-item two-dimenional model.