Derived measures model for obtaining composite and dimension scores

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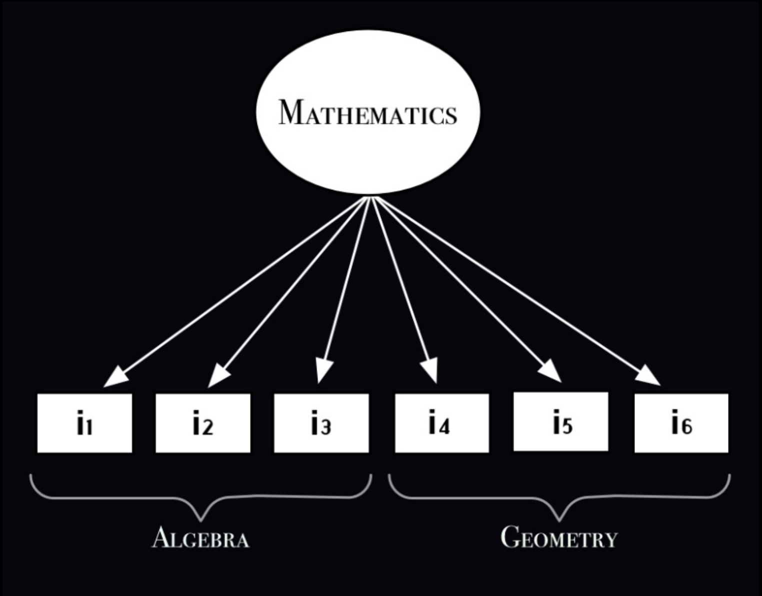
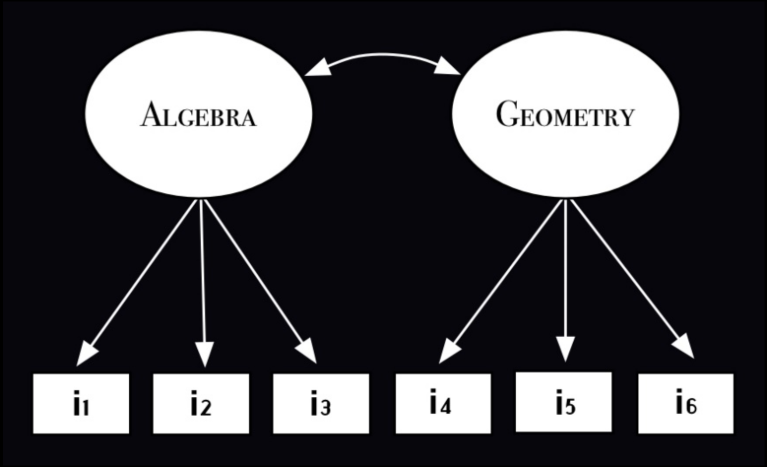
Outline

1. Models with the composite score
   - interpretation of composite and domain scores
2. Derived composite score
   - Interpretation of the derived score
3. Example

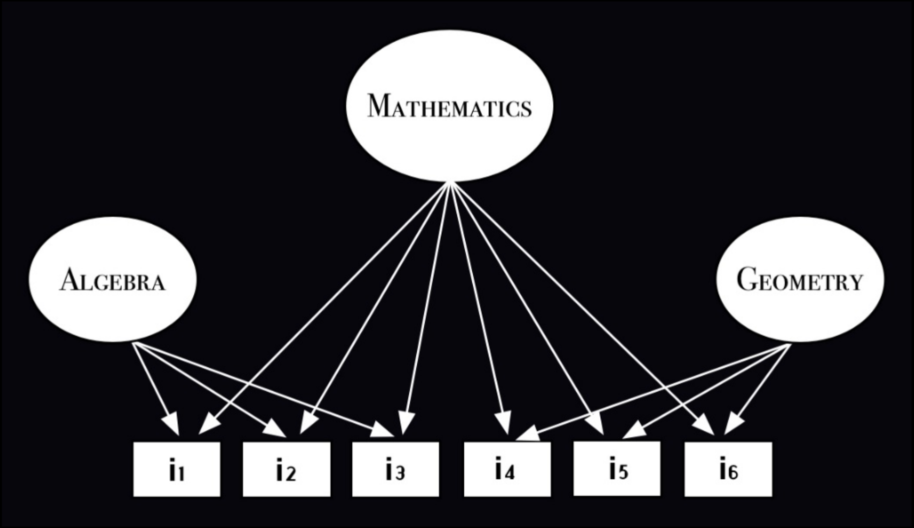
UNIDIMENSIONAL MODEL



MULTIDIMENSIONAL MODEL



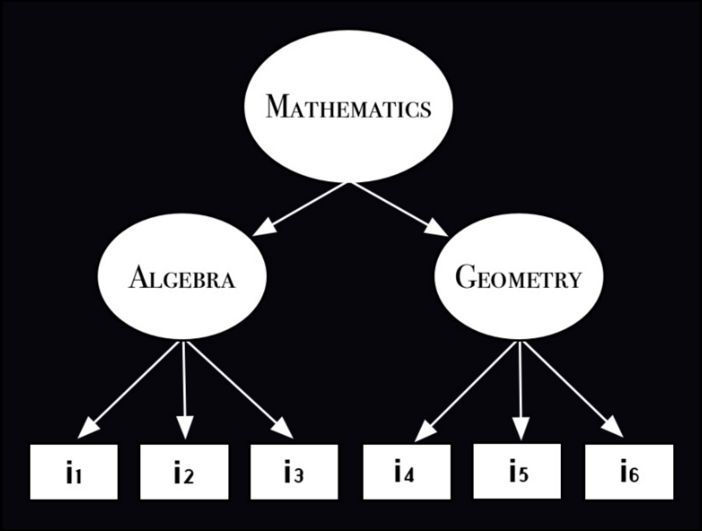
BIFACTOR MODEL



BIFACTOR MODEL

- common factor
  - variance shared by all of the items in the model
- unique factors
  - variance that is unique to items loading on the particular factor
- the common factor is independent of unique factors by definition
  - mathematics ability that is orthogonal to the unique geometry and algebra
  - geometry and algebra factors represent the part of those factors that are orthogonal to the general mathematics factor
    - what is algebra when it is orthogonal to mathematics?
- ideal for testlets

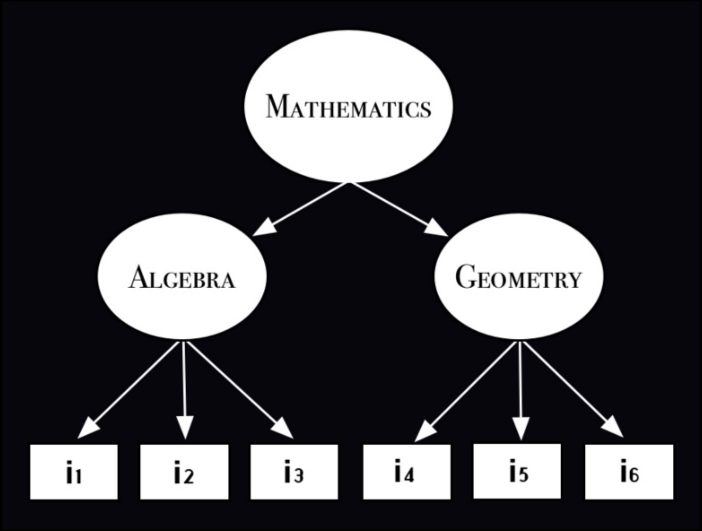
HIGHER-ORDER MODEL



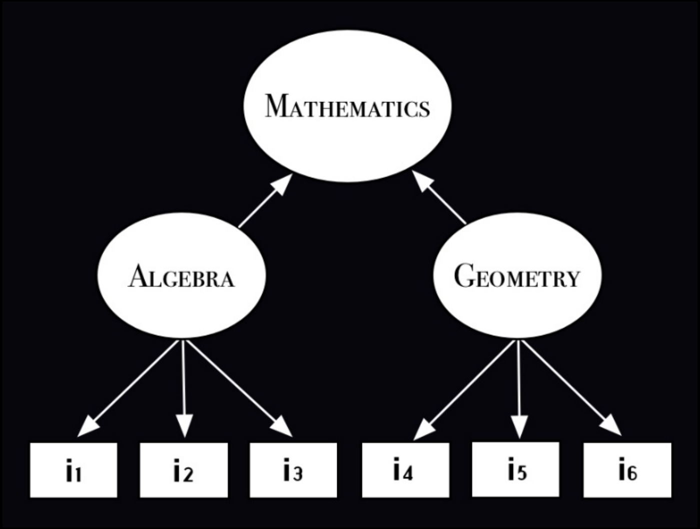
HIGHER-ORDER MODEL

- overall factor
  - extracted from the common variation among first-order factors
  - items are involved indirectly
- domain-specific factors
  - expressed as linear functions of the overall factor
  - assumed to be related implicitly
  - attributed to the overall (second-order) factor
    - correlation between first-order factors is due to the second-order factor
- only between-item multidimensionality

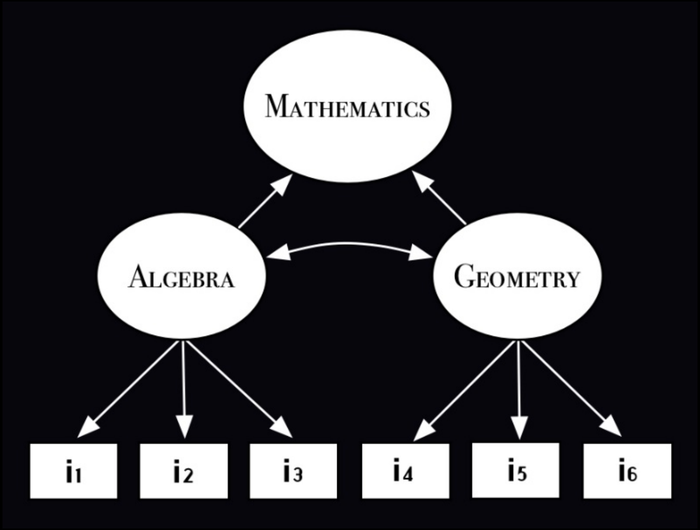
HIGHER-ORDER MODEL



DERIVED MODEL



DERIVED MODEL



DERIVED MODEL

- convenience in interpretation
  - averages across dimensions
- different from simple averaging
  - impose dimensional alignment of dimensions
- plausible values
- flexibility/control over “weights”
  - i.e., (1) equal; (2) proportional to items; (3) length of instruction during the year
- covariance between dimensions can be used
- within-item multidimensionality

DERIVED MODEL

- Dimensional alignment:
  - mean and SD of item parameters across dimensions constrained to be equal
  - this forces common scale across multiple dimensions
  - only then dimensions can be “combined”
  - allows comparison of ability estimates across dimensions

SIMULATION

- Simulated three data sets
  - two-dimensional Rasch model
  - *N*=5000
  - variances = 1; covariance = 0.8
  - Items: *I*=60:
    - data set 1: 30: (-2, 2); 30: (-2, 2)
    - data set 2: 30: (-2, 2); 30: (-2.5, 1.5)
    - data set 3: 30: (-2, 2); 30: (-3, 3)
- Fit unidimensional and derived models

DERIVED MODEL

Data set 1

- [items: 30: (-2, 2); 30: (-2, 2)]
- derived model
  - variances: 0.99 (0.03); 1.00 (0.03)
  - correlation: 0.81
  - means: 0 (fixed); -0.02 (0.01)
  - items: 30: (-2.0, 2.0); 30: (-2.0, 2.0)
  - reliabilities: 0.86; 0.84
- unidimensional model
  - variance: 0.87 (0.02)
  - items: (-2.0, 2.0); (-2.0, 2.0)
  - reliability: 0.90

DERIVED MODEL

Data set 2

- [items: 30: (-2, 2); 30: (-2.5, 1.5)]
- derived model
  - variances: 0.99 (0.03); 1.00 (0.03)
  - correlation: 0.82
  - means: 0 (fixed); 0.48 (0.01)
  - items: 30: (-2.0, 2.0); 30: (-2.0, 2.0)
  - reliabilities: 0.86; 0.81
- unidimensional model
  - variance: 0.87 (0.02)
  - items: (-2.0, 2.0); (-2.4, 1.4)
  - reliability: 0.90

DERIVED MODEL

Data set 3

- [items: 30: (-2, 2); 30: (-3, 3)]
- derived model
  - variances: 1.22 (0.03); 0.78 (0.02)
  - correlation: 0.82
  - means: 0 (fixed); -0.02 (0.01)
  - items: 30: (-2.7, 2.6); 30: (-2.3, 2.3)
  - reliabilities: 0.87; 0.83
- unidimensional model
  - variance: 0.87 (0.02)
  - items: (-2.0, 2.0); (-3.0, 3.0)
  - reliability: 0.89

DERIVED MODEL

Reliability of the derived score

- Gulliksen's reliability formula can be generalized

$$\rho_{dd'} = \frac{\sigma_X^2 \rho_{XX'} + \sigma_Y^2 \rho_{YY'} - 2\rho_{XY}\sigma_X\sigma_Y}{\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y}$$

Thank you!

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